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FORECASTING RESIDENTIAL ELECTRICITY CONSUMPTION IN BRAZIL: APPLICATION OF THE ARX MODEL

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ABSTRACT

This work aims to propose the application of the ARX model to forecast residential electricity consumption in Brazil. Such estimates are critical for decision making in the energy sector, from a technical, economic and environmentally sustainable standpoint. The demand for electricity follows a multiplicative model based on economic theory and involves four explanatory variables: the cost of residential electricity, the actual average income, the inflation of domestic utilities and the electricity consumption. The coefficients of the electricity consumption equation were determined using the ARX model, which considers the influence of exogenous variables to estimate the dependent variable and employs an autoregression process for residual modeling to improve the explanatory power. The resulting model has a determination coefficient of 95.4 percent and all estimated coefficients were significant at the 0.10 descriptive level. Residential electricity consumption estimates were also determined for January and February 2010 within the 95 percent confidence interval, which included the actual consumption figures observed. The proposed model has been shown to be useful for estimating residential electricity consumption in Brazil.

Key-words: Time series. Electricity consumption. ARX modeling.

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PREVISÃO DO CONSUMO RESIDENCIAL DE ENERGIA ELÉTRICA NO BRASIL: APLICAÇÃO DO MODELO ARX

RESUMO

O objetivo deste trabalho é propor a aplicação do modelo ARX para fazer previsões de consumo residencial de energia elétrica no Brasil. Tais previsões são fundamentais para dar suporte às decisões do setor de energia com base técnica, econômica e ambientalmente sustentável. A função da demanda de energia elétrica foi modelada segundo um modelo multiplicativo, com base na teoria econômica, e incluiu quatro variáveis explicativas: tarifa residencial de energia elétrica, rendimento médio real das pessoas ocupadas, índice de inflação de utilidades domésticas e consumo de energia. Para estimar os coeficientes da equação do consumo de energia elétrica foi utilizado o modelo ARX, que considera - na projeção da variável dependente - a influência de variáveis exógenas e modela os resíduos por meio de um processo autorregressivo, a fim de aumentar seu poder explicativo. Os Resultados indicam um coeficiente de determinação de 95,4%. Todos os coeficientes estimados foram estatisticamente significantes num nível descritivo de 0,10. Foram realizadas projeções do consumo residencial de energia elétrica para os meses de janeiro e fevereiro de 2010 no intervalo de confiança a 95%, o qual incluiu os valores verdadeiros de consumo observados. O modelo proposto mostrou ótima performance na previsão do consumo de energia residencial do País.

Palavras-chave: Séries temporais. Energia elétrica. Modelo ARX

1 INTRODUCTION

Decision making in the electricity sector and its consequences for the development of Brazil's infrastructure depends fundamentally on accurate demand estimates. Forecasts covering different timeframes are critical tools to handle the complexity of the management of the domestic electricity system and the day-to-day running of power stations.

The Ministry of Mines and Energy is responsible for devising, articulating and coordinating the country's electricity planning and produces the Decennial Energy Plan encompassing a short-, medium- and long-term view. This is drawn up by EPE - Empresa de Pesquisa Energética (the Energy Research Company). As EPE has stressed, the Decennial Energy Expansion Plan for the 2008-2017 period has important indications that provide guidance for action and decisions on how to balance the country's economic growth estimates, the consequences of electricity needs, and the necessary expansion of supply, based on technical, financial and environmentally sustainable considerations.

Furthermore, according to Miranda (2009), the very short-term forecast, which involves high frequency data, is also essential for reliable and efficient operation of the electrical sector, enabling efficient allocation of electricity, besides indicating possible distortions in upcoming periods (days, hours, or hour fractions).

Concerning the generation of electric energy, Brazil stands out worldwide for its hydroelectric complex, which accounts for 79.6 percent of the country's total installed capacity, while 12.8 percent of total capacity is provided by fossil fuels, according to the 2008-2017 Decennial Energy Plan. On the other hand, according to Goldenberg (2004), approximately 80 percent of all the energy consumed in the world comes from fossil sources and oil products.

Therefore, a quality estimate of future demand for electricity is fundamental in order to manage the energy system and its operating processes better.

Several models have been used to estimate electricity consumption, such as the Box-Jenkins models, the co-integration and distributed lag models, and the time series structural models.

Prior studies have been conducted on the forecast of electricity consumption in Brazil using econometric methodologies. The work of Modiano (1984) was among the first to measure income elasticity and the price of electric energy in Brazil based on annual data from 1963 to 1981. Modiano (1984) used the method of least squares with adjustments for serial correlation by the Cochrane-Orcutt method. In this study, only the long-term income elasticities proved to be elastic in relation to electricity consumption.

Andrade and Lobão (1997) analyzed residential electricity consumption in Brazil based on annual periods from 1963 to 1995, estimating income elasticities and the price of aggregated demand. In a way, they updated the Modiano (1984) study. The work of Andrade and Lobão showed that the short-term income elasticities and price exceeded the long-term ones.

Schmidt and Lima (2004) estimated long-term price elasticities and income using cointegration for the residential, industrial and commercial consumption classes. The estimated income-elasticity obtained was greater than one, whereas the price-elasticity, in module, was lower than one. Demand estimates for 2001 to 2005 were also prepared.

Irffi et al. (2009) estimated electricity demand for the residential, commercial and industrial classes in the Northeast of Brazil from 1970 to 2003, using the DOLS (Dynamic Ordinary Least Squares) method and prepared forecasts for the 2004 to 2010 period. The results obtained were in line with the conclusions of the preceding works found in the domestic literature on the residential, industrial and commercial classes.

Studies conducted in other countries on forecasting electricity consumption involved methods such as unit root tests, DOLS, vector autoregression (VAR), and vector error correction (VEC). Donatos and Mergos (1991) estimated Greek residential demand for electricity, both for the short- and the long-terms, based on data from 1961 to 1986.

In the United States, Silk and Joutz (1997) used the VAR/VEC model to estimate residential demand for electricity, based on annual data on residential electricity consumption from 1949 to 1993, and developed estimates for 1994 and 1995.

Zachariadis and Pashourtidou (2007), in Cyprus, used VEC and unit root tests to estimate the energy consumption in the residential sector and in the service sector. The results indicated that the service sector is less elastic and reverts more quickly to equilibrium than the residential sector.

In the present study, the authors apply the ARX model to estimate electricity consumption in the residential sector in Brazil. ARX considers several exogenous economic variables in the estimate of electricity consumption. Furthermore, it models the residues by means of an autoregression process to increase the model's explanatory power. The study comprised 84 monthly observations, from January 2003 to December 2009.

2 THEORETICAL FRAME OF REFERENCE

2.1 DEMAND FOR ELECTRICITY

The demand for electricity is based on economic theory and is in line with a Cobb-Douglas function, used by Modiano (1984), Andrade and Lobão (1997) and Schmidt (2003). It is a multiplicative and non-linear model, considering the hypotheses described below.

- ✓ Demand is influenced by the mean power tariff charged to residential users, income (average worker income for the residential sector and Gross Domestic Product - GDP for the industrial and commercial sectors), price of products that consume electricity (appliances, for the residential sector, and electricity intensive products for the industrial and commercial sectors), and by the price of an energy source or asset that can replace the original energy supply over time t (the industrial sector being the only one that can have assets that substitute electricity).
- ✓ In the general case, one can describe the demand for electricity as a function of the individual or firm operating at a certain level of activity; thus, it is associated with a cost minimization model.

The function of demand for electricity is given by Equation 2

$$C_t = kP_t^{\alpha} Y_t^{\beta} L_t^{\delta} S_t^{\phi}, k > 0, \alpha < 0, \beta > 0, \delta < 0, \phi > 0 \qquad (2)$$

Applying the natural logarithm, one obtains the following linear equation:

$$\log C_t = \log k + \alpha \log P_t + \beta \log Y_t + \delta \log L_t + \phi \log S_t \qquad (3)$$

where:

- Ct is electricity consumption over time t for the consumption class in question;
- Pt is the tariff (residential, commercial or industrial) for electricity over time t;
- Yt is the average worker income in time t or GDP for the industrial and commercial sectors;
- Lt is the price of home appliances (residential case) or electricity-intensive devices or equipment (for the industrial and commercial cases);
- St is the price of an asset that substitutes electric energy over time t and only applies to the industrial case.

The other parameters are:

- α: price elasticity;
- β: income elasticity;
- δ: price elasticity of the stock of home appliances;
- ϕ: price elasticity of substitute goods;
- k: constant

2.2 ARX MODEL

A simple model to estimate electricity consumption can be obtained using the Box-Jenkins methodology. Here, a time series is estimated by means of the lags of the series itself and of past lagged random errors. This model disregards macroeconomic and microeconomic variables that might affect the dynamics of the demand for electricity.

As an alternative to the Box-Jenkins methodology, the multiple linear regression model estimates electricity consumption by means of several exogenous variables, such as the energy tariff, real income, the inflation of home utilities and temperature, among others.

The ARX model takes into account in the consumption estimate different exogenous variables and it models the remainder using an autoregression model (AR). The advantage of ARX is the adjustment of a multiple linear regression model in conjunction with the adjustment of an autoregression model for the consumption series. The model that explains one dependent variable by means of (a) different exogenous variables, (b) lags of the exogenous variables, and (c) lags of the dependent variable is called the ARX model. The letters AR refer to the lags of the dependent variable, while the letter X is associated with the exogenous variables. The general ARX model is given by Equation (1), below.

$$y_t = \theta_0 + \sum_{j=1}^{p} \theta_j y_{t-j} + \Gamma x_t + \varepsilon_t \qquad (1)$$

where:

y_t : dependent variable over time t and what one means to estimate θ_0 : constant

$x_t = (x_{1t}, x_{1t-1}, ..., x_{1t-n1}, ..., x_{qt}, x_{qt-1}, ..., x_{qt-nq})^t$: vector of exogenous explanatory variables;

$\Gamma = (\Gamma_{1t}, \Gamma_{1t-1}, ..., \Gamma_{1t-n1}, ..., \Gamma_{qt}, \Gamma_{qt-1}, ..., \Gamma_{qt-nq})$: vector of the coefficients of the exogenous variables;

y_{t-j} : dependent variable, with p lags over time, from (t-1) to (t-p);

ε_t : random error, which follows a normal distribution with zero as its mean and a variance given by σw2 and that is normally called 'white noise.'

The variable x(t) is represented by a column matrix whose dimensions are (q x 1) and includes q exogenous variables, which may have specific and different lags amongst them. The matrix line Γ constitutes the coefficients of the exogenous variables x(t).

The model's autoregression part is reflected in p lags of the dependent variable y(t), from (t-1) up to time (t-p). The coefficient for each level of lag of the dependent variable is given by $\Theta_j, \ for \ j = 1,..., p.$

3 RESULTS AND DISCUSSION

3.1 EVOLUTION OF ELECTRICITY CONSUMPTION

The study comprised 84 monthly observations (January 2003 to December 2009) of the consumption of electricity in Brazil's captive market. The consumption series is shown in Figure 1a in GWh, and in Figure 1b, in terms of its natural logarithm.

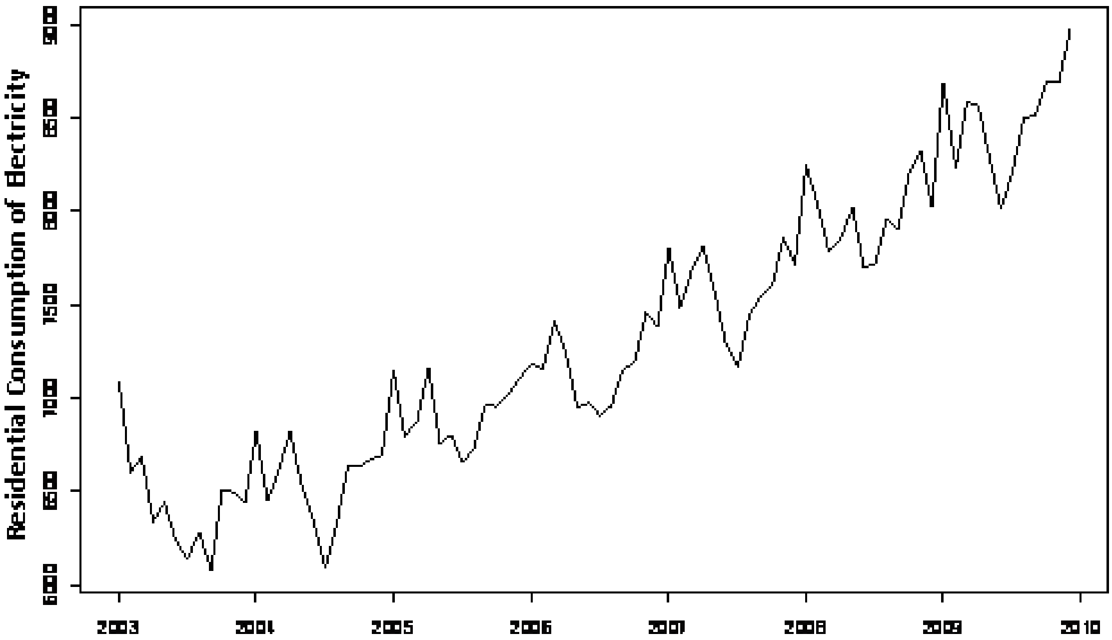

Figure 1a: Evolution of the residential consumption of electricity in Brazil in GWh

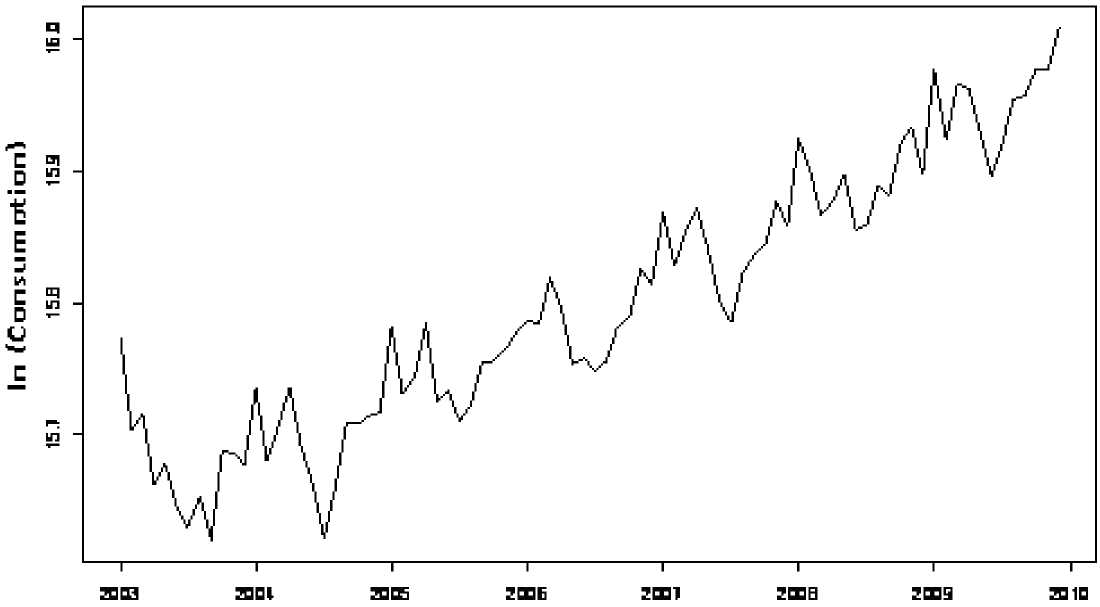

Figure 1b: Evolution of the residential consumption of electricity in Brazil in terms of its natural logarithm (basis of the series given in MWh)

The residential consumption of electricity in Brazil's captive market rose from 77,324 GWh in 2003 to 101,979 GWh in 2009, reflecting mean annual growth of 4.7 percent. Consumption has been rising at a greater rate in the last few years, especially from 2008 to 2009, when it grew 6.5 percent. The commercial sector posted mean annual growth of 5.0 percent between 2003 and 2009, whereas the industrial sector posted a 7.1 percent drop during the same period. In 2003, the residential sector accounted for 32 percent of total consumption, whereas the industrial sector represented 40 percent and the commercial, 20 percent of the total. In 2009, the residential sector increased its share of total consumption to 42 percent and the commercial sector grew to 26 percent, while the industrial sector's share dropped to 31 percent of the total.

Therefore, the residential segment has enjoyed the greatest electricity consumption growth during the period analyzed, due to the growing number of consumers, income expansion, rising sales of electrical appliances and greater mean consumption per family. One should highlight that more than 2 million new consumers were connected to the power grid in 2008, almost half whom lived in the North, Northeast and Midwestern regions. This reflects the expansion of the federal administration's 'Light for Everyone' program.

3.2 ARX MODEL FOR ESTIMATING FUTURE DEMAND FOR ELECTRICITY

As Table 1 shows, four exogenous variables were considered in the study: (a) the residential electricity tariff in R$/MWh, at December 2009 values; (b) the mean actual earnings from work; (c) the domestic utilities inflation index, with December 2009 representing 100; and (d) the actual consumption of electricity, with lagged times.

Table 1: Variables considered in the ARX model

VARIABLE	DEFLATOR	UNIT	SOURCE
Residential consumption of power	-	MWh	ANEEL (Electricity consumption)
Mean residential electricity tariff in constant December 2009 values.	IGP-DI	R$/MWh	ANEEL (Electricity tariff); IPEADATA/FGV (IGP-DI)
Actual mean earnings effectively received by occupied people aged 10 or above, by metropolitan areas	-	R$	IPEADATA/IBGE
IPA-EP of durable consumer goods (home appliances) – Index deflated using the IPA-EP (12/2009=100)	IPA-EP	Index	IPEADATA/IBGE

At first, the ADF (Augmented Dickey-Fuller) test was applied, to test the stationarity of the variables (Table 2). At the level, the null hypothesis of the unit root is maintained for the consumption variables (CO), tariff (I) and inflation of domestic utilities (I), even at a descriptive level of 0.10. For income (R), the null unit root hypothesis is rejected at a 5 percent significance level. However, the null hypothesis is rejected for the four variables in the first and second differences at a descriptive level of 0.01, suggesting that there might be a long-term relation among them.

Table 2: Unit Root Test – Augmented Dickey-Fuller (ADF)

				Critical Values		
Variables	**Level**	**1st Difference**	**2nd Difference**	**1%**	**5%**	**10%**
CO	-0.8546	-12.584	-18.2408	-3.51	2.90	-2.59
T	-0.1075	-7.3318	-15.3275	-3.51	2.90	-2.59
R	-3.4380	-10.8454	-15.4630	-3.51	2.90	-2.59
I	0.5039	-7.4877	-14.7130	-3.51	2.90	-2.59

Based on the multiplicative and non-linear model for modeling electricity consumption, the authors used the logarithm of electricity consumption and of all the exogenous variables, obtaining a linear model. The ARX model was adjusted using the EViews software (version 4).

At first, to develop the multiple linear regression model, the aforementioned 4 exogenous variables were used in time t as well as their lags up to four time points (t-1, t-2, t-3, t-4). The backward method was used to select the exogenous variables, considering, for the exclusion of variables, a descriptive level of 0.10.

The analysis of the remainders of multiple linear regression suggests the adjustment of an autoregression model to the consumption series. Thus, it is necessary to compose an AR model in relation to the multiple regression model obtained previously, giving rise to an ARX model and increasing the developed model's explanation power.

Table 3 shows the coefficients of the adjusted model. One can see that the descriptive level associated with the estimated coefficients is lower than 0.10.

Table 3: Coefficients of the adjusted model

VARIABLES	COEFFICIENTS	STANDARD DEVIATION	T-STUDENT	DESCRIPTIVE LEVEL
C	5179124.00	1286117.00	4.0269	0.0002
T	8735.39	2046.44	4.2686	0.0001
C1	0.19	0.08	2.4386	0.0184
R1	1685.99	187.11	9.0106	0.0000
I1	-46527.00	9022.82	-5.1566	0.0000
R3	516.27	194.98	2.6478	0.0109
R4	558.03	168.10	3.3197	0.0017
AR(4)	-0.24	0.13	-1.7965	0.0786
AR(8)	-0.33	0.12	-2.6888	0.0098
AR(16)	-0.57	0.12	-4.6623	0.0000
AR(20)	-0.34	0.14	-2.4976	0.0159

The equation of the ARX model obtained is:

$$co\_res_t = 5179124{,}0 + 8735{,}4.T + 0{,}19.C1 + 1686{,}0.R1 - 46527{,}0I1 + 516{,}3.R3 + 558{,}0.R4$$
$$- 0{,}24.co\_res_{t-4} - 0{,}33.co\_res_{t-8} - 0{,}57.co\_res_{t-16} - 0{,}34.co\_res_{t-20} + \varepsilon_t$$

where:

- co_res: natural logarithm of the residential consumption of electricity, and:
- co_rest_t: in time t;
- co_rest_{t-4}: in t-4;
- co_rest_{t-8}: in t-8;
- co_rest_{t-16}: in t-16;
- co_rest_{t-20}: in t-20
- T: natural logarithm of the residential tariff of electricity in time t
- C1: natural logarithm of the electricity consumption in t-1;
- R1: natural logarithm of actual mean earnings in t-1;
- R3: natural logarithm of actual mean earnings in t-3;
- R1: natural logarithm of actual mean earnings in t-4;
- I1: natural logarithm of the IPA-EP index of home utilities in time t-1, December 2009 being equal to 100

Figure 2 presents the values adjusted to the demand series with the respective confidence intervals of 95 percent.

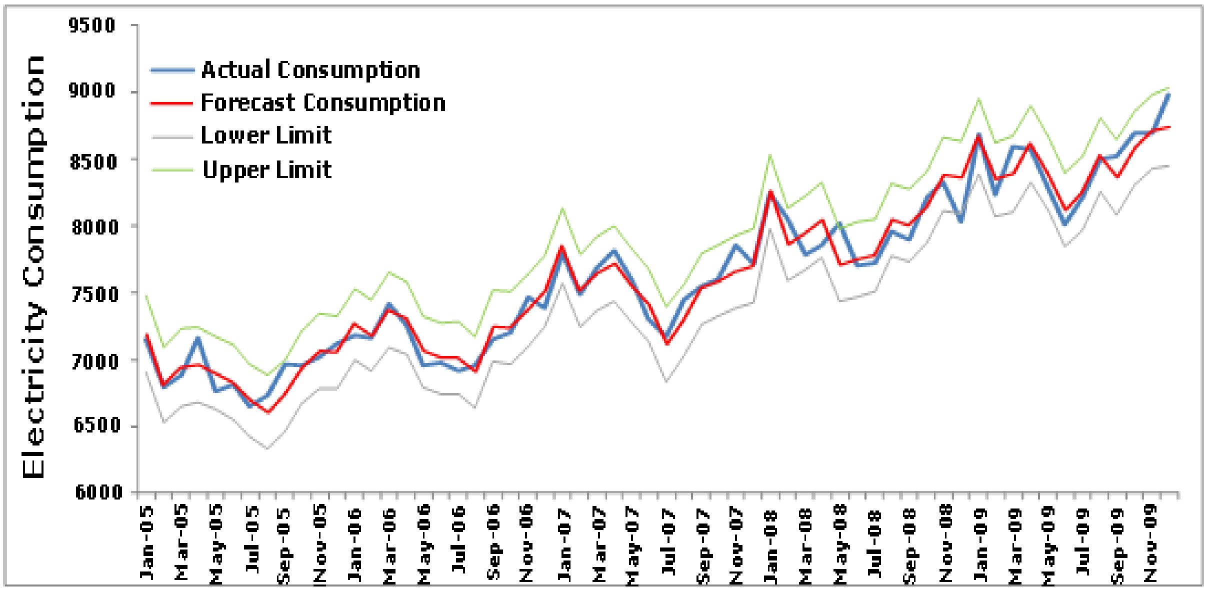

Figure 2: Values adjusted in relation to the demand series with the respective intervals with 95 percent confidence

To estimate the January and February 2010 electricity consumption, the values of the exogenous variables shown in Table 4 were considered. Table 5 shows the estimated values and the 95 percent confidence intervals. One should point out that the real values are included in the 95 percent confidence interval as from the introduction of ARX.

Table 4: Value of the exogenous variables considered for the January and February 2010 forecasts (For IPA-EP, December 2009 = 100)

Month	Residential Tariff (R$/MWh)	Actual Effective Mean Earnings(R$)			IPA-EP of Home Utilities with 1 Month Lag
		1 Month Lag	3 Month Lag	4 Month Lag	
Jan/2010	301.34	1717.86	1371.70	1370.56	100.00
Feb/2010	302.70	1390.99	1457.40	1371.70	99.90

Table 5: Comparison of estimated and actual electricity consumption in January and February 2010

Month	Actual Electricity Consumption (GWh)	Electricity Consumption Estimated by ARX (GWh)		
		Estimated	Lower Limit of IC at 95%	Upper Limit of IC at 95%
Jan/2010	9237	9241	8960	9523
Feb/2010	8933	8767	8467	9067

4 FINAL THOUGHTS

This empirical study aimed to contribute to the modeling of residential consumption of electricity in Brazil using ARX. The study's relevance lies in the need for accurate estimates of the demand for electricity, in order to guide decision-making regarding the management of Brazil's economic growth estimates, the impact on energy requirements and the necessary expansion of supply on a technical, economic and environmentally sustainable basis.

The authors used 84 monthly observations, from January 2003 to December 2009, excluding the effect of the 2001 electricity rationing.

The advantage of using ARX was the adjustment of a linear multiple regression model including relevant exogenous variables based on economic theory, in conjunction with the adjustment of an autoregression model for the consumption series itself.

The model has substantial explanatory power, as evidenced by the determination coefficient of 95.4 percent. All the estimated coefficients were statistically significant at a descriptive level of 0.10.

The estimated electricity consumption based on the ARX models for the months of January and February 2010 were included in the interval at a confidence level of 95 percent.

Extensions of this study include modeling the influence of temperature on residential consumption, by including it in the model. Other extensions of this study might be to develop ARX models to estimate electricity consumption in the industrial segment (using exogenous variables such as actual industry invoicing and the process of goods or assets capable of substituting electricity) and for the commercial segment (including actual retail sales as an exogenous variable), as well as developing forecasts for the next 12 months.

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